in friction factor would be ten to twenty per cent for $Re \gtrsim 30000$.

The metallurgical explanation for the apparent increase in roughness may come from either of two main sources. It is evident that there has been some surface oxidation, but whether the increase in roughness is due to repeated cracking of the protective oxide film (as may occur with nickel–chromium alloys such as Inconel [4]) or whether it may be due to depletion of specific components from preferential attack has not been determined in the present study.

SUMMARY

A scanning electron microscope was used mainly for order-of-magnitude measurements in the present study. Further, only a few measurements were possible due to the short time the equipment was available. However, these limited measurements showed a significant change in roughness dimensions and shape due to heating of Inconel and, perhaps more importantly, demonstrated a new means to measure natural roughness elements smaller than the radius-of-curvature of the typical industrial profilometer. It seems clear that the scanning electron microscope offers considerable potential for more complete characterization of the texture of the surfaces employed in convective and radiative heat transfer applications. By using electronic signal processing, it is likely that useful measurements of the size spectra could be readily recorded and reduced. The improved characterization of the shape is also valuable.

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CORRECTION ON THE LENGTH OF ICE-FREE ZONE IN A CONVECTIVELY-COOLED PIPE

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NOMENCLATURE

Bi, Biot number, hR/k;
h, heat transfer coefficient;
k, thermal conductivity;
Pe, Péclét number $\bar{U}R/\kappa$;
R, pipe radius;
T, temperature;
U, axial velocity;
x, dimensionless axial distance, $X/(Pe\,R)$;
λ, eigenvalue;
Γ, Gamma function;
ϵ, superheat ratio, $(T_{in} - T_f)/(T_f - T_c)$;
κ, thermal diffusivity.

Subscripts
c, external coolant;
e, ice-free length;
f, interface;
in, inlet;
w, water.

Superscript

$\bar{\ }$, average value.

INTRODUCTION

IN REFERENCE [1], the complete set of eigenvalues and eigenfunctions for the well known Graetz problem was presented. By employing an asymptotic solution, the eigenvalues

$$\lambda_n = 4n + \tfrac{8}{3}, n = 0, 1, 2, \ldots \quad (1)$$

were obtained for the case of flow in a round tube with a Dirichlet thermal boundary condition. Good agreement was even found at moderately large value of eigenvalues. Recently, Lock, Freeborn and Nyren [2] applied the same technique to the analysis of ice formation in a convectively-cooled pipe with boundary condition of third kind. The eigenvalues were found from the transcendental equation

$$\frac{1}{Bi_w} + \frac{3^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)\sin\left(\frac{\lambda\pi}{4} - \frac{2\pi}{3}\right)}{2^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\lambda^{\frac{2}{3}}\sin\left(\frac{\lambda\pi}{4} - \frac{\pi}{3}\right)} = 0 \quad (2)$$

which appeared in reference [2] for the analysis of ice-free zone. The length of ice-free zone was predicted from the temperature distribution. Lock *et al.* [2] claimed that the entire solution can be checked for $Bi_w \to \infty$: i.e. a Dirichlet boundary condition.

This note is to point out that it is inaccurate to use the first few eigenvalues obtained from the asymptotic solution to compute the length of ice-free zone for the boundary condition of second or third kind. A conventional power series solution and Crank–Nicolson finite-difference method [3] are employed here to recalculate the length of ice-free zone.

RESULTS AND DISCUSSIONS

The equation subjected to boundary condition of third kind in the analysis of ice-free zone presented in [2] is resolved by the conventional power series solution and by the Crank–Nicolson finite-difference method [3] using mesh size of $\Delta x = \Delta r = 0{\cdot}02$. The new results for variations in length of ice-free zone, x_e, with the superheat ratio, ϵ, are shown in Fig. 1. The solid lines indicate the present power series solution, while individual points are numerical results which agree excellently with the power series solution. The dashed lines are the asymptotic solution of Lock *et al.* [2]. The asymptotic solution checks well with the present ones when the Biot number is large such as 5·0 or 10·0. But the discrepancy becomes more pronounced as the Biot number decreases. It is noted that the difference in the values of x_e between the dashed lines for $Bi_w = 0{\cdot}25$ and $Bi_w = 0{\cdot}5$ is almost the same as the difference in x_e between the dashed lines for $Bi_w = 0{\cdot}5$ and 1·0. This observation contradicts with other group of dashed lines for $Bi_w = 2{\cdot}5$, 5·0 and 10·0.

In order to explain the reason for the discrepancy, comparison between the eigenvalues obtained from the present solution and those from the asymptotic solution is made in the following Table 1.

The eigenvalues for the case of $Bi_w \to \infty$ shown in the last column correspond to the Dirichlet boundary condition. An error of about 1·4 per cent is found for the values of λ_0. The error increases as the Biot number decreases and a difference of 60 per cent is found for the values of λ_0 for the case of $Bi_w = 0{\cdot}25$. Although it is trivial to compute the eigenvalues for the Neumann boundary condition, $Bi_w = 0$, from the physical point of view, it gives a sound reasoning why the error increases as the Biot number decreases.

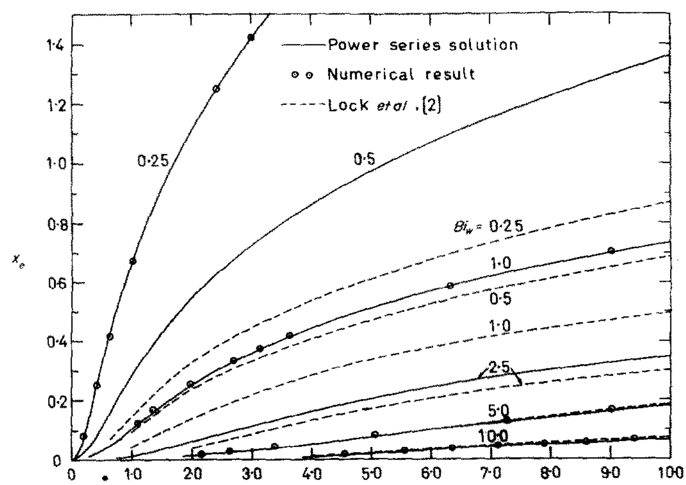

FIG. 1. Length of ice-free zone.

Table 1. Comparison of eigenvalues

	$Bi_w = 0$	0·25	0·5	1·0	2·5	5·0	10·0	∞
λ_0	0·004854	0·9463	1·272	1·641	2·102	2·357	2·517	2·704
	1·333	1·536	1·681	1·878	2·173	2·363	2·496	2·667
λ_1	5·068	5·187	5·295	5·478	5·841	6·135	6·365	6·679
	5·333	5·426	5·509	5·650	5·932	6·172	6·371	6·667
λ_2	9·158	9·234	9·306	9·436	9·729	10·01	10·27	10·67
	9·333	9·399	9·459	9·568	9·814	10·06	10·29	10·67
λ_3	13·20	13·26	13·31	13·42	13·66	13·93	14·20	14·67
	13·33	13·39	13·43	13·52	13·74	13·98	14·22	14·67

The data shown in the upper row are obtained from the present power series solutions and the data in the lower row are obtained by computing the roots of equation (2).

CONCLUSIONS

1. A larger error in the length of ice-free zone results from employing the eigenvalues computed from the asymptotic solution for the boundary condition of second or third kind. Conventional power series method should be used in calculating the first few eigenvalues.

2. The length of ice-free zone predicted by the present power series solution agrees excellently with the numerical result. The previous result [2] checks well with the present solutions when the Biot number is large. But it carries large error when the Biot number is small.

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CONDENSATION OF BINARY MIXTURES OF MISCIBLE VAPORS

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NOMENCLATURE

D, binary diffusion coefficient;
g, acceleration of gravity;
K_L, thermal conductivity of the condensate;
K, constant suction parameter expressing the strength of interfacial suction;
M_1, M_2, molecular weights of the binary species;
P, total pressure;
P_1, P_2, vapor pressure of the pure components [5];
q, actual local surface heat flux evaluated at $(T_i - T_w)$;
q_0, reference heat flux evaluated at $T_i = T_\infty$;